Scientific Report
for the period 01.12.2009 – 30.11.2010

Title of the project: **The study of coupling kinetic equations with stochastic differential equations. Applications to turbulent transport problem in magnetic confined plasmas.**

Partner from Romania: **University of Craiova**
Partner from Belgium: **Universite Libre de Bruxelles**
Period of the project: 01.01.2008 – 30.11.2010

General objective of the project

To study the plasma transport by means of coupling kinetic equations for the plasma particles with stochastic differential equations for the electric and magnetic fields.

Obiective for step IV:

Obiectiv principal:
Coupling dynamics at different time scales and inclusion of non-Gaussian processes

Milestones:
1. Description of the stochastic transitions between two plasma states and coupling between dynamics at different time scales by introducing of a non-stationary stochastic process. (Lect. Dr. Pometescu Nicolae, Assistant research Dr. Babalic Mirela)
2. Modelling of the self organized criticality processes in fusion plasmas. Long range correlation effects**.** (Prof. Dr. Steinbrecher Gyorgy)
3. Clarify some aspects of the anomalous magnetic transport in tokamaks using deterministic models and describe the influence of the stochastic (chaotic) dynamics on the transport properties. (Conf. Dr. Constantinescu Dana)
4. Observe the influence of a stochastic perturbation in deterministic models that exhibit Hopf bifurcation. (Conf. Dr. Constantinescu Dana)
5. Application of the decorrelation trajectory method to the turbulent transport. (Lect. Dr. Negrea Marian, Lect. Dr. Petrisor Iulian)

Actions in step IV:

Mobility action was accomplished by Lect. Dr. Nicolae Pometescu at Universite Libre de Bruxelles in the period 19-24 July 2010. In this mobility period was studied the suggestion of Dr Nicolae Pometescu to use a non-stationary stochastic process in order to introduce multi-time different scales to model the transport process in plasma by solving the differential stochastic equation of V-Langevin kind.

Conf. Dr Dana Constantinescu accomplished a mobility action at Universite Libre de Bruxelles in the period 29 October – 3 November 2010. In this period were analysed opportunities to study transport in plasma by using fractional methods.
From Belgium, Dr Boris Weyssow visited University of Craiova in the period 3-9 November 2010 where he discussed with Dr Pometescu Nicolae and Dr Mirela Babalic on the model of particle transport in plasma using non-stationary stochastic process. Also he worked with Dr Negrea Marian and Dr Petrisor Iulian on the numerical simulation for the study of the transport in MHD plasma in test-particle approximation.
Dr Weyssow Boris and Dr Constantinescu Dana worked on the influence of stochastic perturbation on systems that exhibit Hopf bifurcation.

The web site of the project: http://cis01.central.ucv.ro/proiectecercetare/valonia/ was update.

Scientific description

The turbulence plays an important role in plasma deconfining and transfer of the energy between particles and instabilities.
The experimental studies of the fusion plasma show that particles exhibit a non-diffusive behaviour (anomalous transport). This kind of transport can be studied in terms of stochastic differential equations (as Langevin equations) or in terms of Self Organized Criticality models.
The evaluation of running diffusion coefficients is an important goal in fusion plasma physics as it provides a first insight into the transport properties of charged particles in electro-magnetic fields. The set of stochastic velocity equations, the so-called V-Langevin equations, were used by many authors to describe a trace particle transport in fluctuating magnetic field. In fusion plasma this model can be applied to some physical situation in which the plasma oscillate between two temperatures in rather stochastic manner. One example can be found in a study of the impact of large ELMs on JET for unfueled 3.0 MA H-mode discharge.
Other example we met in pellet fuelling of Ohmic and Lower Hybrid driven discharges in Tore Supra. The experimental results show for electrons in the central region a transition between two temperatures, almost regular in the Ohmic case, and almost random in the Lower Hybrid case, on the scale of a few seconds.
Transport of charged particles in guiding centre approximation is described in terms of Langevin stochastic differential equations where the stochastic additive term is stationary [1,2] or non-stationary [3].
It was already shown that different tools to model some aspects of transport (deterministic, stochastic, fractional) are linked between them, in that they can be derived from the same conservation law in integral form: this unifies the description of transport in a strong background field (Vlasov equation), with collisions (Fokker-Plank equation) and under the interaction with intermittent, turbulent electric fields (fractional transport equation in velocity space). For this reason it is important to study mixed models (for example stochastic

differential equations or fractional differential equations) in order to obtain a realistic description of transport phenomena.

1. **Description of the stochastic transitions between two plasma states and coupling between dynamics at different time scales by introducing of a non-stationary stochastic process.** (Lect. Dr. Pometescu Nicolae, Asistent cercetare Dr. Babalic Mirela)

Using Langevin equations we perform an analysis of the transport of the guiding centers of charge particles along and perpendicularly to the main magnetic field. The model includes collisions and therefore a thermal velocity which we relate to the plasma temperature evolution through a stochastic process, and a magnetic field represented by a different stochastic process which is considered independent of the thermal velocity.

1.1. The V-Langevin equations

The general form of the V-Langevin equations have been derived previously in the form

$$\frac{d\vec{x}_\perp}{dt} = \beta \vec{b}\left[\vec{x}_\perp, z\right] V_z(t)$$

$$\frac{dz}{dt} = V_z(t)$$

$$\frac{dV_z}{dt} = -\upsilon_z V_z(t) + \alpha_z(t)$$

where $V_z(t)$ is the stochastic parallel velocity, $\vec{b}[\vec{x}_\perp, z] = \vec{B}/B$ is the unit vector along the equilibrium magnetic field, β is the amplitud of the magnetic fluctuations, υ_z is the collision frequency and α_z is the stochastic acceleration in parallel direction.

When α_z is a white noise process we obtained

$$<\alpha_z(t)>= 0 \; , \; <\alpha_z(t)\; \alpha_z(u)>= A\delta(t-u)$$

In our previous study [1, 2] acceleration α_z was defined as a coloured noise process,

$$\alpha_z(t) = \frac{1}{b-a}\left[b-\eta(t)\right]\alpha_{za}(t) + \frac{1}{a-b}\left[a-\eta(t)\right]\alpha_{zb}(t)$$

where the stochastic process $\eta(t)$, with one of the values *a* or *b*, is defined as a stationary stochastic process with master equations

$$\partial_t P(a,t\,|\,x,t_0) = -\lambda P(a,t\,|\,x,t_0) + \mu P(b,t\,|\,x,t_0)$$

$$\partial_t P(b,t\,|\,x,t_0) = \lambda P(a,t\,|\,x,t_0) - \mu P(b,t\,|\,x,t_0)$$

In the present study the stochastic process $\eta(t)$ is non-stationary and defined by master equations

$$\partial_t P(a,t \mid x,t_0) = -\lambda(t)P(a,t \mid x,t_0) + \mu(t)P(b,t \mid x,t_0)$$
$$\partial_t P(b,t \mid x,t_0) = \lambda(t)P(a,t \mid x,t_0) - \mu(t)P(b,t \mid x,t_0)$$

with

$$\lambda(t) = \omega \sin^2(\omega t/2)$$
$$\mu(t) = \omega \cos^2(\omega t/2)$$

As in the stationary case

$$\lambda + \mu = \frac{1}{\tau_0}$$

where τ_0 is the time correlation. Also we have

$$<\eta(t)\ \alpha_{zj}(t)>=0\,,$$
$$<\alpha_{zi}(t)>=0\ ,\ <\alpha_{zi}(t)\ \alpha_{zj}(u)>=2\delta_{ij}\upsilon_i V_{T_i}^2(x_\perp)\delta(t-u)$$

In this case we obtain the conditional probabilities

$$P(a,t \mid \eta,t_0) = \frac{1}{2}[1+F(t)] + \left\{\frac{1}{2}[1-F(t_0)]\delta_{a,\eta_0} - \frac{1}{2}[1-F(t_0)]\delta_{b,\eta_0}\right\}\exp[-\omega(t-t_0)]$$
$$P(b,t \mid \eta,t_0) = \frac{1}{2}[1-F(t)] - \left\{\frac{1}{2}[1-F(t_0)]\delta_{a,\eta_0} - \frac{1}{2}[1-F(t_0)]\delta_{b,\eta_0}\right\}\exp[-\omega(t-t_0)]$$

where

$$F(t) = \frac{1}{\sqrt{2}}\sin(\omega t + \pi/4)$$

The asymptotic behaviour, when $t \to \infty$, is given as

$$P_{asimp}(a,t \mid \eta,t_0) = \frac{1}{2}[1+F(t)]$$
$$P_{asimp}(b,t \mid \eta,t_0) = \frac{1}{2}[1-F(t)]$$

The mean value read as

$$<\eta(t)>=\frac{a+b}{2}+\frac{a-b}{2}F(t)+\left\{\eta_0-\left[\frac{a+b}{2}+\frac{a-b}{2}F(t_0)\right]\right\}\exp[-\omega(t-t_0)]$$

with the asymptotic oscillate value

$$<\eta(t)>_{asimp}=\frac{a+b}{2}+\frac{a-b}{2}F(t)$$

The second order correlation is obtained as

$$<\eta(t)\eta(u)>=N_1(t,u)+N_2(u)\exp[-\omega|t-u|]+N_3(t,t_0)\exp[-\omega(u-t_0)]+N_4(u,t_0)\exp[-\omega(t-t_0)]$$

where

$$N_1(t,u)=\left(\frac{a+b}{2}\right)^2+\frac{a^2-b^2}{4}[F(u)+F(t)]+\left(\frac{a-b}{2}\right)^2 F(t)F(u)$$

$$N_2(u)=\left(\frac{a-b}{2}\right)^2\left[1-F^2(u)\right]$$

$$N_3(t,t_0)=\frac{1}{4}\left\{2\eta_0^2[1-F(t)F(t_0)]-2b\eta_0F(t)[1-F(t_0)]+2a\eta_0F(t)[1+F(t_0)]\right.$$
$$\left.-\left(a^2+b^2\right)-\left(a^2-b^2\right)[F(t)+F(t_0)]-\left(a^2+b^2\right)F(t)F(t_0)\right\}$$

$$N_4(u,t_0)=\frac{1}{4}\left\{\eta_0^2[1-F(t_0)][1-F(u)]+\eta_0^2[1+F(t_0)][1+F(u)]+2b\eta_0[1-F(t_0)]F(u)\right.$$
$$\left.-2a\eta_0[1+F(t_0)]F(u)-a^2[1+F(t_0)][1-F(u)]-b^2[1-F(t_0)][1+F(u)]\right\}$$

The asymptotic behaviour is

$$<\eta(t)\eta(u)>_{asimp}=\left(\frac{a+b}{2}\right)^2+\frac{a^2-b^2}{4}[F(u)+F(t)]+\left(\frac{a-b}{2}\right)^2 F(t)F(u)$$
$$+\left(\frac{a-b}{2}\right)^2\left[1-F^2(u)\right]\exp\left[-\frac{|t-u|}{\tau_0}\right]$$

For the stochastic process α_z , the mean value is zero

$$<\alpha_z(t)>=0\ ,$$

and the second order correlation in asymptotic limit is

$$<\alpha_z(t)\ \alpha_z(u)>=\frac{1}{2}\upsilon_{za}V_{Ta}^2\left\{1+F(t)+F(u)+F(t)F(u)+\left[1-F^2(u)\right]\exp[-\omega|t-u|]\right\}\delta(t-u)$$
$$+\frac{1}{2}\upsilon_{zb}V_{Tb}^2\left\{1-F(t)-F(u)+F(t)F(u)+\left[1-F^2(u)\right]\exp[-\omega|t-u|]\right\}\delta(t-u)$$

1.2 Magnetic field model

The magnetic field is assumed to be fluctuating according to a stochastic law defined by:

$$\vec{B}=B_0\left\{\vec{e}_z+\beta b_x(z)\vec{e}_x+\beta b_y(z)\vec{e}_y\right\}$$

The parameter $\beta<<1$ is a dimensionless number measuring the characteristic amplitude of the fluctuations, relative to the equilibrium magnetic field. The quantities $b_x(z)$, $b_y(z)$ are dimensionless and have an arbitrary variation on space and time (in general case). Here we consider magnetic fluctuation time-independent (frozen turbulence) with gyrotropic symmetry (cylindrical symmetry).

The stochastic process for the components $b_x(z)$, $b_y(z)$ is described by the second order correlations,

$$<b_m(z)b_n(z')>=\delta_{mn}\exp\left[-\frac{(z-z')^2}{2\lambda_{\|}}\right]$$

Using the Fourier representation of the magnetic field we obtain,

$$< \hat{b}_m(k)\hat{b}_n(k') >= \hat{\mathcal{B}}_{\|}(k)\,\delta(k+k')\,\delta_{mn}$$

where

$$\hat{\mathcal{B}}_{\|}(k)=\frac{\lambda_{\|}}{\sqrt{2\pi}}\exp\left(-\lambda_{\|}^2 k^2/2\right)$$

1.3 Second order correlation of the parallel velocity

The second order correlation read as

$$\begin{aligned}\langle \mathrm{v}_z(t)\,\mathrm{v}_z(t+\tau)\rangle &= \mathrm{v}_{0z}^2\exp\left[-\nu_z(2t+\tau)\right]\\ &+\left(\mathrm{v}_{Ta}^2\nu_{za}+\mathrm{v}_{Tb}^2\nu_{zb}\right)\exp\left[-\nu_z(2t+\tau)\right]\frac{1}{2\nu_z}\left(\exp\left[2\nu_z t\right]-1\right)\\ &+\left(\mathrm{v}_{Ta}^2\nu_{za}-\mathrm{v}_{Tb}^2\nu_{zb}\right)\exp\left[-\nu_z(2t+\tau)\right]\\ &\times\left[\frac{\omega_0-2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}+\exp\left[2\nu_z t\right]\left[-\frac{\omega_0-2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\cos\left[\omega_0 t\right]+\frac{\omega_0+2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\sin\left[\omega_0 t\right]\right]\right]\end{aligned}$$

with the asymptotic behaviour

$$\begin{aligned}\langle \mathrm{v}_z(t)\,\mathrm{v}_z(t+\tau)\rangle_{asym} &= \mathrm{v}_{Ta}^2\nu_{za}\left[\frac{1}{2\nu_z}-\frac{\omega_0-2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\cos\left[\omega_0 t\right]+\frac{\omega_0+2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\sin\left[\omega_0 t\right]\right]\exp\left[-\nu_z\tau\right]\\ &+\mathrm{v}_{Tb}^2\nu_{zb}\left[\frac{1}{2\nu_z}+\frac{\omega_0-2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\cos\left[\omega_0 t\right]-\frac{\omega_0+2\nu_z}{2\left(4\nu_z^2+\omega_0^2\right)}\sin\left[\omega_0 t\right]\right]\exp\left[-\nu_z\tau\right]\end{aligned}$$

In the particular case $\omega = 2\nu_z$ we obtain

$$< V_z(t)V_z(t+\tau) >_{asimp}=\frac{\nu_{za}V_{Ta}^2}{\nu_z}\left(\frac{1}{2}+\frac{\sin(2\nu_z t)}{4}\right)\exp(-\nu_z\tau)+\frac{\nu_{zb}V_{Tb}^2}{\nu_z}\left(\frac{1}{2}-\frac{\sin(2\nu_z t)}{4}\right)\exp(-\nu_z\tau)$$

Conclusions

The second order correlation of the parallel velocity has in the asymptotic limit an oscillate variation with an amplitude attenuate by an exponential factor.

The explicit form of the oscillation depends on the relation between the two frequencies: the collision frequency and the external frequency which break the correlation.

The leading results of this study were presented at the conference [3], Physics Conference TIM-10, Timisoara, 25 - 27 November 2010, TCP-O11.

2. Modelling of the self organized criticality processes in fusion plasmas. Long range correlation effects (Prof. Dr. Steinbrecher Gyorgy)

2.1. Generalized linear analytic model of instability growth

Continuous time, one-dimensional affine stochastic evolution equations were studied both in physical and mathematical literature ([4-9]). This interest in ASEE comes partly from the occurrence of heavy tail (HT) in the stationary probability distribution functions (PDF), in the models including the self-organized criticality models of the fusion or space plasma turbulence [4]. Despite their apparent formal simplicity, even in the classical examples of the discrete time ASEE, the affine iterated function systems, the stationary cumulative probability function in one or two-dimensional case has complicated fractal structure and is currently used for image compression.

The apparently simple continuous time, one-dimensional ASEE from Ref. [4] captures important qualitative features of the very complex dynamics controlled fusion devices. It explains the simultaneous occurrence of the very small value of the heavy tail exponent and the large correlation time, approximate self-similarity, of the driving multiplicative noise. An important conclusion from [4] is that even a small amplitude driving noise, but having large correlation time, produces a highly delocalized stationary PDF, with small HT exponent.

This phenomenon is a typical stochastic effect; it is an intermediate effect between negative and positive Liapunov exponents in the systems without noise.

The ASEE model equation that is considered here is a class of one-dimensional random differential equation, which extends previous results from ref [4], by using new topological vector space methods. The additive and the multiplicative random terms in our model are stationary processes. The multiplicative term is a generalization of the stationary Gaussian processes, having very general correlation function. Our results and those from Ref. [7, 8] are complementary.

Our approach is also focused on the generalization of the previous model [4], by including more general driving noises and some restricted class of nonlinear term.

We studied the general one-dimensional quasi-linear equation

$$\frac{d\mathbf{X}_{\omega}(t)}{dt} = A_{\omega}(t)\mathbf{X}_{\omega}(t) + \mathbf{B}_{\omega}(t;\mathbf{X}_{\omega}(t))$$

where $A_{\omega}(t)$ is a stationary random process with negative mean value and algebraically decaying correlation function. The non linear term is a stochastic process, $\mathbf{B}_{\omega}(t;\mathbf{X}_{\omega}(t))$ having finite moments of sufficiently large order. For large amplitudes of X(t) the term $\mathbf{B}_{\omega}(t;\mathbf{X}_{\omega}(t))$ is supposed to remain finite.

The following results were obtained

a. Under very general conditions we proved that there exists a stationary probability distribution function. The convergence to the stationary distribution function is described by a class of weak topologies in the space of probability measures that characterises the distribution of the amplitude X(t). These topologies are defined by suitable Lebesgue spaces with non standard exponent: the convergence is expressed in the term of L^p spaces that are allowed to have non-standard values for the exponent p i.e. $0<p<1$.
b. The heavy tail exponent of the stationary probability distribution function was explicitly computed. We proved that it is completely determined by the statistical properties of the leading linear term.
c. The convergence results obtained extend largely previous results on one- dimensional linear stochastic equations from Refs. [4-9]. The use of the non-compact version of the Kakutani –Stone theorem concerning the density of a lattice of generalized Holder continuous functions on a local compact metric space give a sound framework to the study of anomalous transport phenomena in turbulent plasma.

2.2. **Numerical modelling of self-organized criticality models of the fusion and space plasma turbulence**

For the numerical simulation of the effects of the long-range correlations in the edge plasma turbulence, an algorithm for generating superdiffusive fractional Brownian motion (fBm) was elaborated [11]. The algorithm uses a new class of representation, presented also in Ref. [4], obtained from a self-similar ensemble of Ornstein-Uhlenbeck processes and allows a straightforward implementation on parallel computers.

The typical trajectories of fBm are continuous but are nowhere differentiable, and give a rich class of examples of functions that are non-monotone, even restricted to arbitrary small domain. Nevertheless fBm is considered one of the must successful stochastic models for natural and economical processes

The special place of fBm in mathematical physics is related to the fact that it is uniquely defined (modulo three parameters) by simple axiom [12] and its occurrence in the study of the fusion plasmas is explained by functional central limit theorems [12]

The fBm is a straightforward, logical, generalization of the classical Brownian motion. As a consequence of the previous simple axiom, it is expected that according to the central limit theorem, fBm appears as a realistic model for self-similar stationary processes having non-trivial correlations of the increments.

In physical applications the persistent fBm related to the impurity transport in random electromagnetic field, the persistent fBm appears as a scaling limit of more complicated stationary transport processes as proved in refs [13-14].

The persistent fBm appears also as a hidden noise that triggers the instabilities in self organized criticality models [4, 11]. This aspect is related to the first exit time statistics problem, where unfortunately there are few analytic results and numerical methods must be used. This was one the main motivation of this study.

It was proved in a rigorous style the conjecture exposed in ref. [4]: the persistent fBm can be approximated in suitable Hilbert space metric by a linear combination of self-similar ensemble of integrated Ornstein-Uhlenbeck processes. Rigorous and efficient control of the approximation errors was elaborated. The algorithm allows including correction due to the cut-of of the integrated Ornstein-Uhlenbeck processes with very short correlation times. Was elaborated also an optimised correction of the errors resulting by the truncation of the components with short correlation times.

The generation method is easy to adapt to parallel computers. Contrary to the standard generation method that uses fast Fourier transform, this method can be used to the study of noise driven processes in very large time scales, that are not known at the start of the simulations.

These results will be used in the acceleration to the existing numerical methods in the integrated tokamak modelling

3. **Integrability versus chaos in non-autonomous Hamiltonian systems. Applications to the study of some transport phenomena** (Conf. Dr. Constantinescu Dana)

The phase space of some Hamiltonian systems is a complex mixture of invariant zones whose points have regular, respectively chaotic dynamics. The regular zones (where the system is almost integrable) are characterized by a reduced transport. Such zones act sometimes as transport barriers which separate different chaotic zones. Inside the chaotic zone the transport is increased, due to the mixing properties of the system.

We proposed general results concerning the existence and the localization of internal transport barriers for Hamiltonian systems with periodical perturbation in 1 ½ degrees of freedom. We systematically studied the influence of the parameters which define the unperturbed Hamiltonian on the transport properties.

The results were applied for the study of the magnetic transport in tokamaks where the equations of the magnetic field lines can be written in a Hamiltonian form:

- It was proved that a transport barrier exists in the low shear region for enough small perturbations of the ideal system; it was shown the reversed-shear is not a mandatory condition for the existence of the transport barrier.
- It was proved that the transport barrier intersects the so called “regular curve” which can be analytically derived from the model. It was shown that the regular curve and the shearless curve (which can be only numerically derived) are closed, so the result we obtained can be directly applied for locating the transport barriers. It was shown that the transport barrier does not exist if all the points of the regular curve have chaotic (stochastic) behaviour. This result gives a criterion for obtaining a globally stochastic behaviour of the system which drastically modifies the transport properties.

4**. The influence of stochastic perturbation on systems that exhibits Hopf bifurcation** (Conf. Dr. Constantinescu Dana)

A deterministic three-dimensional system of differential equations which exhibits Hopf bifurcation was studied. The obtained results are interesting for fusion plasma physics because the system models the global changes of the plasma parameters during a discharge in tokamak (typical changes in profiles or profile gradients) in the presence local instabilities. In the system two different time scales of underlying dynamics were considered and some oscillations of saw-tooth type were pointed out.

The existence of a stable limit cycle (due to the Hopf bifurcation) significantly influences the dynamics of the system, inducing a regular oscillating behavior associated with periodicity.

The study of the corresponding stochastic system obtained by the perturbation of the bifurcation parameter by a Gaussian white noise started. From the numerical simulations it was observed that, from the point of view of random dynamical systems, the deterministic Hopf bifurcation was destroyed: for all values of the bifurcation parameter the system has a unique global random attractor which gives a different perspective on the asymptotical behavior of the system.

The work is in progress because analytical results must be obtained in order to explain the numerically observed features of the bifurcation behavior under the parametric white noise perturbation.

5. **Application of the decorrelation trajectory method to the turbulent transport**. (Lect. Dr. Negrea Marian, Lect. Dr. Petrisor Iulian)

We have calculated using the decorrelation trajectory method and the numerical simulations the diffusion coefficients for electrons. In order to do that, the Langevin stochastic equations, specific to the electrostatic turbulence combined with the stochastic sheared anisotropic magnetic field were studied for specific models. Conditions for the “strange” regimes (subdiffusion and superdiffusion) were also included and the trapping phenomena, which are responsible for the subdiffusion, were emphasized. The results concerning the diffusion coefficients obtained by the decorrelation trajectory method were compared with those corresponding to the numerical calculation [16, 17]. Typical trajectory obtained by numerical simulations is represented in Figure 1.

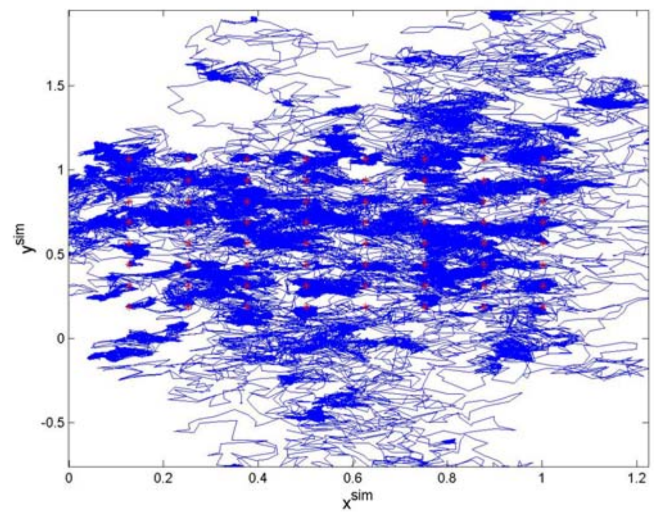

Figure1. Typical trajectory

We have also analyzed the test particle diffusion in an electromagnetic stochastic field and we have considered that both the electric and magnetic fields are stochastic with different correlation lengths and time [18]. We have considered in the model that the electrostatic potential and the magnetic potential are stochastic but stochastically uncorrelated. This kind of analysis is fundamental for gaining a proper understanding of many hot plasma experiments that operate naturally in such conditions or use external coils to generate electromagnetic stochasticity. Only the auto-correlations were taken into account for future studies. This statement was verified by using the TURBO code developed at Universite Libre de Bruxelles. We investigated (with TURBO) two-dimensional turbulence produced into a box of 512 x 512 modes by taking ab initio the kinematical viscosity equals to the magnetic diffusivity. We solved the full MHD equations by choosing some different type of forcing. The different but fixed energy and helicity injection rates in a shell of wave vectors were considered in order to evaluate the correlation tensors. We have concluded that for the electric and magnetic potentials the order of magnitude of the cross-correlations is much smaller than the auto-correlations. In Figure 2 the MSD in radial and poloidal directions were represented for a coupling parameter $\alpha= t_A/t_p =100$, where t_A is the Alfven time and t_p (the proper time of the particle) is the inverse of the particle's Larmor frequency. The trajectory is represented in Figure 3 for the same coupling parameter. The strange transport is obvious. Thus, this conclusion will be used as starting point for further studies like test particle movement in various combination of the stochastic fields. The other system of equations was prepared for the numerical study: the movement of the test particle in a stochastic electromagnetic field that is introduced in the problem as a solution of the magneto-hydrodynamics equations. A friction force, which is considered as a deterministic quantity and proportional with the relative velocity, will be added to the Lorentz force expression.

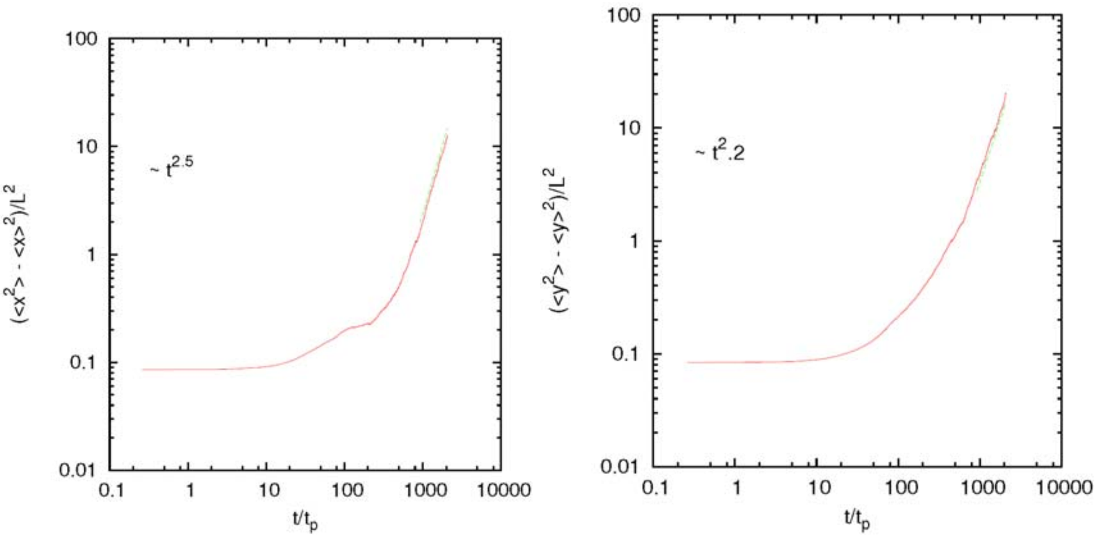

Figure 2 - The poloidal (left) and radial (right) mean square displacements.

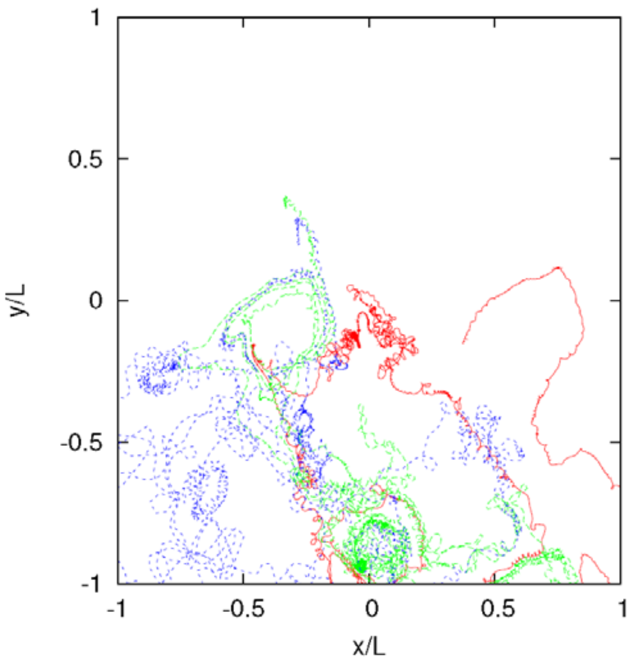

Figure 3 - Trajectory of the test particle for $\alpha = 100$

We have used partly the computer facilities of ULB-VUB, Belgium, in collaboration with Dr. D. Carati, Dr. Bogdan Teaca and the research group from ULB, Belgium.

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